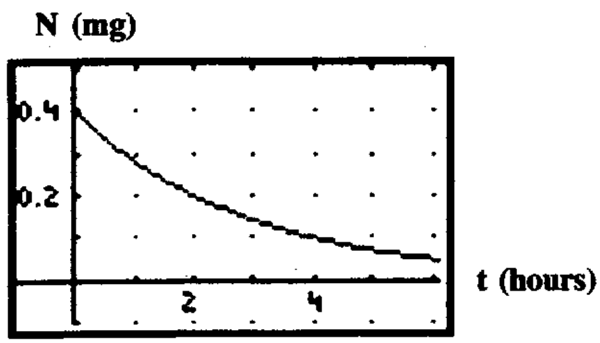
Interpreting Functions (Sections 1.1 and 1.2) **Date:** ____________

The topic of a **function** is one of the major concepts in any College Algebra course. Recall that it is used to represent the dependence of one quantity upon another. A complete understanding of functions, and function notation, is necessary to be successful in any Calculus course.

Example #1

Nicotine Problem: The graph shown below represents the amount of nicotine, $N = f(t)$, in mg, in a person's bloodstream as a function of the time t, in hours, since the person finished smoking a cigarette.



a. Estimate f(3) and interpret it in terms of the problem.

b. About how many hours have passed before the nicotine level is down to 0.1 mg? Write this result in function notation.

c. What is the vertical intercept? What does it represent in terms of nicotine?

d. If this function had a horizontal intercept, what would it represent?

Example #2

Fahrenheit/Celsius Temperatures: According to the Arizona Daily Star, the highest recorded temperature in the United States on Monday, August 17th, 2009, was 113° F (or 45° C) in Death Valley CA, and the coldest temperature was 23° F (or -5° C) in West Yellowstone, MT. Knowing that there is a linear relationship between a Celsius temperature and the corresponding Fahrenheit temperature,

a. Write an equation that shows how the Celsius temperature C depends on the Fahrenheit temperature F.

b. Interpret the slope of the line in part a above.

c. If the high temperature in San Fancisco that day was 20° C, what was the corresponding Fahrenheit temperature?

Example #3

Product Promotion: A cereal company finds that the number of people who will buy one of its products in the first month that it is introduced is linearly related to the amount of money it spends on advertising. If it spends $40,000 on advertising, then 100,000 boxes of cereal will be sold, and if it spends $60,000, then 180,000 boxes will be sold.

a. Write an equation describing the relation between the amount A spent on advertising and the number N of boxes sold.

The independent variable is: ________ The dependent variable is: ________

b. Interpret the slope of the line in part a above.

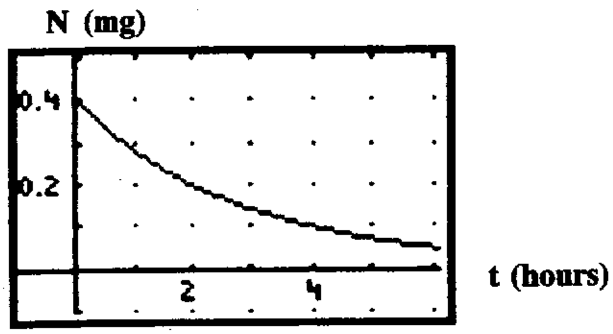
c. How much advertising is needed to sell 300,000 boxes of cereal?

Rates of Change (Section 1.3)

Date: ____________

Last class, we wrote the equation of a line that represented the relationship between a Celsius temperature and a Fahrenheit temperature. A linear function has a <u>constant rate of change</u> which is the ____________________.
But, how do we calculate a rate of change for a function that is not linear?

Let's revisit the function given in the Nicotine Problem, shown below.



a. Estimate the change in the amount of nicotine over the first 6 hours. ____________________

b. Estimate the average hourly rate at which the nicotine is decreasing over the first 6 hours.
Note: The word "average" is used because, as we will see, the rate of change can vary within the interval.

c. Does it make sense that this value is negative? Explain.

d. Estimate the average rate of change of the amount of nicotine left in the body between $t = 2$ and $t = 6$.

> In general, if y is a function of t, so $y = f(t)$, then the average rate of change of y between $t = a$ and $t = b$ is:
>
> Average rate of change = ____________________

Back to the Nicotine Problem.

e. What basic type of function might best represent the graph of this function? ____________

f. Find a function in the form $y = a \cdot b^t$, that could be used to model this function.

(Hint: We need to find values for a and b.) Check your answer by graphing the function on your calculator.

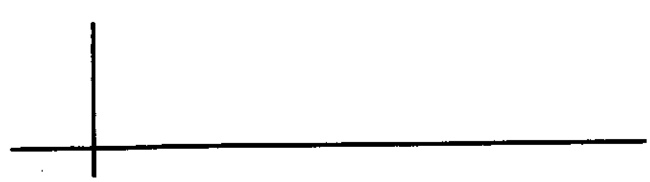
g. Use your function, not the graph, and calculate the average rate of change of nicotine between $t = 1.5$ and $t = 3.5$.

Definitions:

Example: Given the function $y = f(x) = \sqrt{x+1}$.

a. Find the average rate of change of f between $x = 0$ and $x = 3$.

b. Sketch the graph of $y = f(x)$, and represent the average rate of change as the slope of a line.



c. Which is larger, the average rate of change of the function between $x = 0$ and $x = 3$, or $x = 3$ and $x = 6$? What does this tell you about the graph of the function?

Definitions:

Examples:

1. Given the graph of the function below.

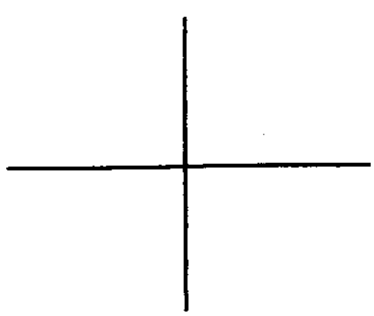
Estimate the interval(s) over which the function is:

a. Increasing ______________ b. Decreasing ______________

c. Concave Up ______________ d. Concave Down: ______________

e. Estimate the average rate of change between $t = 1$ and $t = 3$. ______________

2. Draw a circle below with center at the origin.



Identify a section (quadrant) of the circle that is:

a. decreasing and concave up. ________ b. increasing and concave down. ________

c. increasing and concave up. ________ d. decreasing and concave down. ________

3. Values of 5 functions, F(t), G(t), H(t), I(t), and J(t) are shown in the chart below.

t	F(t)	G(t)	H(t)	I(t)	J(t)
10	15	15	45	60	15
20	22	18	36	57	17
30	28	21	30	52	20
40	33	24	26	44	24
50	37	27	23	33	29

Identify the function whose graph would be:

a. linear. ____________

b. decreasing and concave up. ____________

c. increasing and concave down. ____________

d. increasing and concave up. ____________

e. decreasing and concave down. ____________

Distance, Velocity, and Speed

A ball is thrown up in the air. The height, y, of the ball above the ground first increases, and then decreases. It's height t seconds after it is thrown is shown in the chart below.

t (sec)	0	0.5	1	1.5	2	2.5	3	3.5	4
y (feet)	5	36	59	74	81	80	71	54	29

1. What is the change in the ball's height during the first 3 seconds? ____________________

2. What is the average rate of change of the height of the ball during the first 3 seconds? ____________________

Note: The average rate of change of height with respect to time is the ______________________________

3. Find the average velocity of the ball over the time interval from t = 2 to t = 4. Explain the sign of the answer.

Note: There is a difference between <u>velocity</u> and <u>speed</u>. Speed is the <u>magnitude of velocity</u>, so it is always positive.